OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE
MATHEMATICS (MEI) 4757
Further Applications of Advanced Mathematics (FP3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Option 1: Vectors

1 Four points have coordinates

$$A\,(3,\ 8,\ 27), \quad B\,(5,\ 9,\ 25), \quad C\,(8,\ 0,\ 1) \quad \text{and} \quad D\,(11,\ p,\ p),$$

where p is a constant.

(i) Find the perpendicular distance from C to the line AB. **[5]**

(ii) Find $\overrightarrow{AB} \times \overrightarrow{CD}$ in terms of p, and show that the shortest distance between the lines AB and CD is

$$\frac{21|p-5|}{\sqrt{17p^2 - 2p + 26}}.$$ **[8]**

(iii) Find, in terms of p, the volume of the tetrahedron ABCD. **[4]**

(iv) State the value of p for which the lines AB and CD intersect, and find the coordinates of the point of intersection in this case. **[7]**

Option 2: Multi-variable calculus

2 In this question, L is the straight line with equation $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$, and $\mathrm{g}(x,\ y,\ z) = (xy + z^2)\mathrm{e}^{x-2y}$.

(i) Find $\dfrac{\partial \mathrm{g}}{\partial x}$, $\dfrac{\partial \mathrm{g}}{\partial y}$ and $\dfrac{\partial \mathrm{g}}{\partial z}$. **[4]**

(ii) Show that the normal to the surface $\mathrm{g}(x,\ y,\ z) = 3$ at the point $(2,\ 1,\ -1)$ is the line L. **[4]**

On the line L, there are two points at which $\mathrm{g}(x,\ y,\ z) = 0$.

(iii) Show that one of these points is P $(0,\ 3,\ 0)$, and find the coordinates of the other point Q. **[4]**

(iv) Show that, if $x = -2\mu$, $y = 3 + 2\mu$, $z = \mu$, and μ is small, then

$$\mathrm{g}(x,\ y,\ z) \approx -6\mu\mathrm{e}^{-6}.$$ **[3]**

You are given that h is a small number.

(v) There is a point on L, close to P, at which $\mathrm{g}(x,\ y,\ z) = h$. Show that this point is approximately

$$\left(\tfrac{1}{3}\mathrm{e}^6 h,\ 3 - \tfrac{1}{3}\mathrm{e}^6 h,\ -\tfrac{1}{6}\mathrm{e}^6 h\right).$$ **[2]**

(vi) Find the approximate coordinates of the point on L, close to Q, at which $\mathrm{g}(x,\ y,\ z) = h$. **[7]**

Option 3: Differential geometry

3 A curve C has equation $y = x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}$, for $x \geqslant 0$.

(i) Show that the arc of C for which $0 \leqslant x \leqslant a$ has length $a^{\frac{1}{2}} + \frac{1}{3}a^{\frac{3}{2}}$. **[5]**

(ii) Find the area of the surface generated when the arc of C for which $0 \leqslant x \leqslant 3$ is rotated through 2π radians about the x-axis. **[5]**

(iii) Find the coordinates of the centre of curvature corresponding to the point $\left(4, -\frac{2}{3}\right)$ on C. **[9]**

The curve C is one member of the family of curves defined by

$$y = p^2 x^{\frac{1}{2}} - \frac{1}{3}p^3 x^{\frac{3}{2}} \quad \text{(for } x \geqslant 0\text{)},$$

where p is a parameter (and $p > 0$).

(iv) Find the equation of the envelope of this family of curves. **[5]**

Option 4: Groups

4 The group $F = \{p, q, r, s, t, u\}$ consists of the six functions defined by

$$\mathrm{p}(x) = x \qquad \mathrm{q}(x) = 1 - x \qquad \mathrm{r}(x) = \frac{1}{x} \qquad \mathrm{s}(x) = \frac{x-1}{x} \qquad \mathrm{t}(x) = \frac{x}{x-1} \qquad \mathrm{u}(x) = \frac{1}{1-x},$$

the binary operation being composition of functions.

(i) Show that st = r and find ts. **[4]**

(ii) Copy and complete the following composition table for F. **[3]**

	p	q	r	s	t	u
p	p	q	r	s	t	u
q	q	p	s	r	u	t
r	r	u	p	t	s	q
s	s	t	q	u	r	p
t	t	s	u			
u	u	r	t			

(iii) Give the inverse of each element of F. **[3]**

(iv) List all the subgroups of F. **[4]**

The group M consists of $\left\{1, -1, e^{\frac{\pi}{3}j}, e^{-\frac{\pi}{3}j}, e^{\frac{2\pi}{3}j}, e^{-\frac{2\pi}{3}j}\right\}$ with multiplication of complex numbers as its binary operation.

(v) Find the order of each element of M. **[4]**

The group G consists of the positive integers between 1 and 18 inclusive, under multiplication modulo 19.

(vi) Show that G is a cyclic group which can be generated by the element 2. **[3]**

(vii) Explain why G has no subgroup which is isomorphic to F. **[1]**

(viii) Find a subgroup of G which is isomorphic to M. **[2]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 In this question, give probabilities correct to 4 decimal places.

An electronic control unit on an aircraft is inspected weekly, replaced if necessary, and is labelled A, B, C or D according to whether it is in its first, second, third or fourth week of service.

In Week 1, the unit is labelled A.

At the start of each subsequent week, the following procedure is carried out.

When the unit is labelled A, B or C, it is tested; if it passes the test it is relabelled B, C or D respectively; if it fails the test it is replaced by a new unit which is labelled A.

When the unit is labelled D, it is replaced by a new unit which is labelled A.

The probability that a unit fails the test is 0.16 when it is labelled A, 0.28 when it is labelled B, and 0.43 when it is labelled C.

This situation is modelled as a Markov chain with four states.

(i) Write down the transition matrix. **[2]**

(ii) In Week 10, find the probability that the unit is labelled C. **[3]**

(iii) Find the week (apart from Week 1) in which the probabilities that the unit is labelled A, B, C, D first form a decreasing sequence. Give the values of these probabilities. **[3]**

(iv) Find the probability that the unit is labelled B in Week 8 and is labelled C in Week 16. **[4]**

(v) Following a week in which the unit is labelled D, find the expected number of consecutive weeks in which the unit is labelled A. **[2]**

(vi) Find the equilibrium probabilities that the unit is labelled A, B, C or D. **[4]**

An airline has 145 of these units installed in its aircraft. They are all subjected to the inspection procedure described above, and may be assumed to behave independently.

(vii) In the long run, find how many of these units are expected to be replaced each week. **[2]**

A different manufacturer has now been chosen to make the units. The inspection procedure remains the same as before, but the probabilities that the unit fails the test have changed. The equilibrium probabilities that the unit is labelled A, B, C or D are now found to be 0.4, 0.25, 0.2 and 0.15 respectively.

(viii) Find the new probabilities that the unit fails the test when it is labelled A, B or C. **[4]**

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RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **4757**

Further Applications of Advanced Mathematics (FP3)

Mark Scheme for June 2010

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1 (i)	$\overrightarrow{AC}\times\overrightarrow{AB}=\begin{pmatrix}5\\-8\\-26\end{pmatrix}\times\begin{pmatrix}2\\1\\-2\end{pmatrix}=\begin{pmatrix}42\\-42\\21\end{pmatrix}$	B2	Give B1 for one component correct
	Perpendicular distance is $\dfrac{\lvert\overrightarrow{AC}\times\overrightarrow{AB}\rvert}{\lvert\overrightarrow{AB}\rvert}$	M1	
	$=\dfrac{\sqrt{42^2+42^2+21^2}}{\sqrt{2^2+1^2+2^2}}=\dfrac{63}{3}$	M1	Calculating magnitude of a vector product
	$=21$	A1 **5**	www
	OR $\left[\begin{pmatrix}3+2\lambda\\8+\lambda\\27-2\lambda\end{pmatrix}-\begin{pmatrix}8\\0\\1\end{pmatrix}\right].\begin{pmatrix}2\\1\\-2\end{pmatrix}=0$ M1 A1 $2(2\lambda-5)+(\lambda+8)-2(-2\lambda+26)=0$ A1 ft $\lambda=6$ [F is $(15,\ 14,\ 15)$] $\text{CF}=\sqrt{7^2+14^2+14^2}=21$ M1A1		Appropriate scalar product
(ii)	$\overrightarrow{AB}\times\overrightarrow{CD}=\begin{pmatrix}2\\1\\-2\end{pmatrix}\times\begin{pmatrix}3\\p\\p-1\end{pmatrix}=\begin{pmatrix}3p-1\\-2p-4\\2p-3\end{pmatrix}$	B1 B1 B1	
	$\overrightarrow{AC}.(\overrightarrow{AB}\times\overrightarrow{CD})=\begin{pmatrix}5\\-8\\-26\end{pmatrix}.\begin{pmatrix}3p-1\\-2p-4\\2p-3\end{pmatrix}$	M1	
	$=5(3p-1)-8(-2p-4)-26(2p-3)$ [$=-21p+105$]	A1 ft	
	$\lvert\overrightarrow{AB}\times\overrightarrow{CD}\rvert=\sqrt{(3p-1)^2+(-2p-4)^2+(2p-3)^2}$ $=\sqrt{17p^2-2p+26}$	B1 ft	
	Distance is $\dfrac{\lvert\overrightarrow{AC}.(\overrightarrow{AB}\times\overrightarrow{CD})\rvert}{\lvert\overrightarrow{AB}\times\overrightarrow{CD}\rvert}=\dfrac{21\lvert p-5\rvert}{\sqrt{17p^2-2p+26}}$	M1A1 (ag) **8**	Correctly obtained
(iii)	$V=(\pm)\ \frac{1}{6}(\overrightarrow{AC}\times\overrightarrow{AB}).\overrightarrow{AD}=(\pm)\ \frac{1}{6}\begin{pmatrix}42\\-42\\21\end{pmatrix}.\begin{pmatrix}8\\p-8\\p-27\end{pmatrix}$	M1 A1 ft	Appropriate scalar triple product In any form
	$=(\pm)\ 56-7(p-8)+\frac{7}{2}(p-27)$	M1	Evaluation of scalar triple product *Dependent on previous M1*
	$=(\pm)\ \frac{35}{2}-\frac{7}{2}p$ $=\frac{7}{2}\lvert p-5\rvert$	A1 **4**	$\frac{1}{6}(105-21p)$ or better
(iv)	Intersect when $p=5$	B1	
	$\begin{pmatrix}3\\8\\27\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}=\begin{pmatrix}8\\0\\1\end{pmatrix}+\mu\begin{pmatrix}3\\5\\4\end{pmatrix}$	B1 ft M1	Equations of both lines *(may involve p)* Equation for intersection *(must have different parameters)*
	$3+2\lambda=8+3\mu$	A1 ft	Equation involving λ and μ
	$8+\lambda=5\mu$ [$8+\lambda=p\mu$] $27-2\lambda=1+4\mu$ [$27-2\lambda=1+(p-1)\mu$]	A1 ft	Second equation involving λ and μ *or* Two equations in $\lambda,\ \mu,\ p$
	$\lambda=7,\quad \mu=3$	M1	Obtaining λ or μ
	Point of intersection is $(17,\ 15,\ 13)$	A1 **7**	

2 (i)	$\dfrac{\partial g}{\partial x} = (y + xy + z^2)\,e^{x-2y}$ $\dfrac{\partial g}{\partial y} = (x - 2xy - 2z^2)\,e^{x-2y}$ $\dfrac{\partial g}{\partial z} = 2z\,e^{x-2y}$	M1 A1 A1 A1 **4**	Partial differentiation
(ii)	At $(2, 1, -1)$, $\dfrac{\partial g}{\partial x} = 4$, $\dfrac{\partial g}{\partial y} = -4$, $\dfrac{\partial g}{\partial z} = -2$ Normal has direction $\begin{pmatrix} 4 \\ -4 \\ -2 \end{pmatrix}$ *L* passes through $(2, 1, -1)$ and has this direction	M1 A1 M1 A1 (ag) **4**	
(iii)	When $g = 0$, $xy + z^2 = 0$ $(2-2\lambda)(1+2\lambda) + (-1+\lambda)^2 = 0$ $3 - 3\lambda^2 = 0$ $\lambda = \pm 1$ $\lambda = 1$ gives $P(0, 3, 0)$ $\lambda = -1$ gives $Q(4, -1, -2)$	 M1 M1 A1 (ag) A1 **4**	Obtaining a value of λ Or B1 for verifying $g(0, 3, 0) = 0$ *and* showing that P is on *L*
(iv)	At P, $\dfrac{\partial g}{\partial x} = 3e^{-6}$, $\dfrac{\partial g}{\partial y} = 0$, $\dfrac{\partial g}{\partial z} = 0$ $\delta g \approx \dfrac{\partial g}{\partial x}\delta x + \dfrac{\partial g}{\partial y}\delta y + \dfrac{\partial g}{\partial z}\delta z$ $= 3e^{-6}(-2\mu) + 0 + 0 = -6\mu e^{-6}$	M1 M1 A1 (ag) **3**	OR give M2 A1 www for $g(-2\mu,\ 3+2\mu,\ \mu)$ $= (-3\mu^2 - 6\mu)\,e^{-6\mu-6} \approx -6\mu\,e^{-6}$
(v)	When $-6\mu\,e^{-6} \approx h$, $\mu \approx -\frac{1}{6}e^{6}h$ Point $(-2\mu,\ 3+2\mu,\ \mu)$ is approximately $(\frac{1}{3}e^{6}h,\ 3 - \frac{1}{3}e^{6}h,\ -\frac{1}{6}e^{6}h)$	M1 A1 (ag) **2**	
(vi)	At Q, $\dfrac{\partial g}{\partial x} = -e^{6}$, $\dfrac{\partial g}{\partial y} = 4e^{6}$, $\dfrac{\partial g}{\partial z} = -4e^{6}$ When $x = 4 - 2\mu$, $y = -1 + 2\mu$, $z = -2 + \mu$ $\delta g \approx (-e^{6})(-2\mu) + (4e^{6})(2\mu) + (-4e^{6})(\mu)$ $= 6\mu\,e^{6}$ If $6\mu\,e^{6} \approx h$, then $\mu \approx \frac{1}{6}e^{-6}h$ Point is approximately $(4 - \frac{1}{3}e^{-6}h,\ -1 + \frac{1}{3}e^{-6}h,\ -2 + \frac{1}{6}e^{-6}h)$	M1 M1 M1A1 M1 A2 **7**	 OR give M1 M2 A1 www for $g(4-2\mu,\ -1+2\mu,\ -2+\mu)$ $= (-3\mu^2 + 6\mu)\,e^{-6\mu+6} \approx 6\mu\,e^{6}$ Give A1 for one coordinate correct *If partial derivatives are not evaluated at Q, max mark is M0M1M0M0*

3 (i)	$\dfrac{dy}{dx}=\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{\frac{1}{2}}$	B1	
	$1+\left(\dfrac{dy}{dx}\right)^2=1+(\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{\frac{1}{2}})^2$	M1	
	$=1+\frac{1}{4}x^{-1}-\frac{1}{2}+\frac{1}{4}x=\frac{1}{4}x^{-1}+\frac{1}{2}+\frac{1}{4}x$		
	$=(\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{\frac{1}{2}})^2$	A1	
	Arc length is $\int_0^a(\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{\frac{1}{2}})\,dx$		
	$=\left[\,x^{\frac{1}{2}}+\frac{1}{3}x^{\frac{3}{2}}\,\right]_0^a$	M1	
	$=a^{\frac{1}{2}}+\frac{1}{3}a^{\frac{3}{2}}$	A1 (ag) **5**	
(ii)	Curved surface area is $\int 2\pi y\,ds$	M1	For $\int y\,ds$
	$=\int_0^3 2\pi(\,x^{\frac{1}{2}}-\frac{1}{3}x^{\frac{3}{2}}\,)(\,\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{\frac{1}{2}}\,)\,dx$	A1	Correct integral form *including limits*
	$=2\pi\int_0^3(\,\frac{1}{2}+\frac{1}{3}x-\frac{1}{6}x^2\,)\,dx$		
	$=2\pi\left[\,\frac{1}{2}x+\frac{1}{6}x^2-\frac{1}{18}x^3\,\right]_0^3$	M1A1	For $\frac{1}{2}x+\frac{1}{6}x^2-\frac{1}{18}x^3$
	$=3\pi$	A1 **5**	
(iii)	When $x=4$, $\dfrac{dy}{dx}=-\dfrac{3}{4}$	B1	
	Unit normal vector is $\begin{pmatrix}-\frac{3}{5}\\-\frac{4}{5}\end{pmatrix}$	M1 A1 ft	Finding a normal vector Correct unit normal (either direction)
	$\dfrac{d^2y}{dx^2}=-\frac{1}{4}x^{-\frac{3}{2}}-\frac{1}{4}x^{-\frac{1}{2}}\quad(=-\dfrac{5}{32})$	B1	
	$\rho=\dfrac{\{\,1+(-\frac{3}{4})^2\,\}^{\frac{3}{2}}}{(-)\,\frac{5}{32}}\quad(=\dfrac{125/64}{5/32}=\dfrac{25}{2})$	M1 A1 ft	Applying formula for ρ or κ
	$\mathbf{c}=\begin{pmatrix}4\\-\frac{2}{3}\end{pmatrix}+\dfrac{25}{2}\begin{pmatrix}-\frac{3}{5}\\-\frac{4}{5}\end{pmatrix}$	M1	
	$=\begin{pmatrix}-3\frac{1}{2}\\-10\frac{2}{3}\end{pmatrix}$	A1 A1 **9**	
(iv)	Differentiating partially w.r.t. p	M1	
	$0=2px^{\frac{1}{2}}-p^2x^{\frac{3}{2}}$	A1	
	$p=\dfrac{2}{x}$		
	Envelope is $y=\dfrac{4}{x^2}x^{\frac{1}{2}}-\dfrac{1}{3}\dfrac{8}{x^3}x^{\frac{3}{2}}$	M1 A1	
	$y=\dfrac{4}{3}x^{-\frac{3}{2}}$	A1 **5**	

4 (i)	$\mathrm{s\,t}(x) = \mathrm{s}\left(\dfrac{x}{x-1}\right) = \dfrac{\dfrac{x}{x-1}-1}{\dfrac{x}{x-1}}$ $= \dfrac{x-(x-1)}{x} = \dfrac{1}{x} = \mathrm{r}(x)$ $\mathrm{t\,s}(x) = \mathrm{t}\left(\dfrac{x-1}{x}\right) = \dfrac{\dfrac{x-1}{x}}{\dfrac{x-1}{x}-1}$ $= \dfrac{x-1}{(x-1)-x} = 1-x = \mathrm{q}(x)$	M1 A1 (ag) M1 A1 **4**	
(ii)	$\begin{array}{c\|cccccc} & \mathrm{p} & \mathrm{q} & \mathrm{r} & \mathrm{s} & \mathrm{t} & \mathrm{u} \\ \hline \mathrm{p} & \mathrm{p} & \mathrm{q} & \mathrm{r} & \mathrm{s} & \mathrm{t} & \mathrm{u} \\ \mathrm{q} & \mathrm{q} & \mathrm{p} & \mathrm{s} & \mathrm{r} & \mathrm{u} & \mathrm{t} \\ \mathrm{r} & \mathrm{r} & \mathrm{u} & \mathrm{p} & \mathrm{t} & \mathrm{s} & \mathrm{q} \\ \mathrm{s} & \mathrm{s} & \mathrm{t} & \mathrm{q} & \mathrm{u} & \mathrm{r} & \mathrm{p} \\ \mathrm{t} & \mathrm{t} & \mathrm{s} & \mathrm{u} & \mathbf{q} & \mathbf{p} & \mathbf{r} \\ \mathrm{u} & \mathrm{u} & \mathrm{r} & \mathrm{t} & \mathbf{p} & \mathbf{q} & \mathbf{s} \end{array}$	B3 **3**	Give B2 for 4 correct, B1 for 2 correct
(iii)	Element: p, q, r, s, t, u Inverse: p, q, r, u, t, s	B3 **3**	Give B2 for 4 correct, B1 for 2 correct
(iv)	$\{\mathrm{p}\},\ F$ $\{\mathrm{p},\ \mathrm{q}\},\ \{\mathrm{p},\ \mathrm{r}\},\ \{\mathrm{p},\ \mathrm{t}\}$ $\{\mathrm{p},\ \mathrm{s},\ \mathrm{u}\}$	B1B1B1 B1 **4**	*Ignore these in the marking* Deduct one mark for each non-trivial subgroup in excess of four
(v)	Element: 1, -1, $e^{\frac{\pi}{3}j}$, $e^{-\frac{\pi}{3}j}$, $e^{\frac{2\pi}{3}j}$, $e^{-\frac{2\pi}{3}j}$ Order: 1, 2, 6, 6, 3, 3	B4 **4**	Give B3 for 4 correct, B2 for 3 correct B1 for 2 correct
(vi)	$2^1=2,\ 2^2=4,\ 2^3=8,\ 2^4=16,\ 2^5=13,\ 2^6=\mathbf{7}$ $2^7=14,\ 2^8=9,\ 2^9=\mathbf{18},\ 2^{10}=17,\ 2^{11}=15,\ 2^{12}=11$ $2^{13}=3,\ 2^{14}=6,\ 2^{15}=12,\ 2^{16}=5,\ 2^{17}=10,\ 2^{18}=1$ Hence 2 has order 18	M1 A1 A1 **3**	Finding (at least two) powers of 2 For $2^6=7$ and $2^9=18$ Correctly shown *All powers listed implies final A1*
(vii)	*G* is abelian (so all its subgroups are abelian) *F* is not abelian	B1 **1**	*Can have 'cyclic' instead of 'abelian'*
(viii)	Subgroup of order 6 is $\{1,\ 2^3,\ 2^6,\ 2^9,\ 2^{12},\ 2^{15}\}$ i.e. $\{1,\ 7,\ 8,\ 11,\ 12,\ 18\}$	M1 A1 **2**	or B2

Pre-multiplication by transition matrix

5 (i)	$\mathbf{P}=\begin{pmatrix}0.16 & 0.28 & 0.43 & 1\\ 0.84 & 0 & 0 & 0\\ 0 & 0.72 & 0 & 0\\ 0 & 0 & 0.57 & 0\end{pmatrix}$	B2 **2**	*Allow tolerance of* ± 0.0001 *in probabilities throughout this question* Give B1 for two columns correct
(ii)	$\mathbf{P}^9\begin{pmatrix}1\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0.3349\\0.3243\\0.2231\\0.1177\end{pmatrix}$ Prob$(C)=0.2231$	M2 A1 **3**	Using $\mathbf{P}^9$ Give M1 for using $\mathbf{P}^{10}$
(iii)	Week 5 $\mathbf{P}^4\begin{pmatrix}1\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0.5020\\0.2851\\0.1577\\0.0552\end{pmatrix}$	B1 M1 A1 **3**	First column of a power of **P** *SC* Give B0M1A1 for Week 9 and 0.3860 0.3098 0.2066 0.0976
(iv)	$\mathbf{P}^7=\begin{pmatrix}. & . & . & .\\ 0.2869 & . & . & .\\ . & . & . & .\\ . & . & . & .\end{pmatrix}$ $\mathbf{P}^8=\begin{pmatrix}. & . & . & .\\ . & . & . & .\\ . & 0.2262 & . & .\\ . & . & . & .\end{pmatrix}$ Probability is 0.2869×0.2262 $=0.0649$	M1M1 M1 A1 **4**	Elements from $\mathbf{P}^7$ and $\mathbf{P}^8$ Multiplying appropriate probabilities
(v)	Expected run length is $\dfrac{1}{1-0.16}=1.19$ (3 sf)	M1 A1 **2**	Allow 1.2
(vi)	$\mathbf{P}^n\ \rightarrow\begin{pmatrix}0.3585 & 0.3585 & 0.3585 & 0.3585\\ 0.3011 & 0.3011 & 0.3011 & 0.3011\\ 0.2168 & 0.2168 & 0.2168 & 0.2168\\ 0.1236 & 0.1236 & 0.1236 & 0.1236\end{pmatrix}$ A: 0.3585 B: 0.3011 C: 0.2168 D: 0.1236	M1 M1 A2 **4**	Evaluating $\mathbf{P}^n$ with $n\geq 10$ *or Obtaining (at least) 3 equations from* $\mathbf{P}\mathbf{p}=\mathbf{p}$ Limiting matrix with equal columns *or Solving to obtain one equilib prob* Give A1 for two correct
(vii)	Expected number is 145×0.3585 ≈ 52	M1 A1 ft **2**	
(viii)	$\begin{pmatrix}a & b & c & 1\\ 1-a & 0 & 0 & 0\\ 0 & 1-b & 0 & 0\\ 0 & 0 & 1-c & 0\end{pmatrix}\begin{pmatrix}0.4\\0.25\\0.2\\0.15\end{pmatrix}=\begin{pmatrix}0.4\\0.25\\0.2\\0.15\end{pmatrix}$ $0.4a+0.25b+0.2c+0.15=0.4$ $0.4(1-a)=0.25$ $0.25(1-b)=0.2$ $0.2(1-c)=0.15$ $a=0.375,\quad b=0.2,\quad c=0.25$	M1 A1 M1 A1 **4**	Transition matrix and $\begin{pmatrix}0.4\\0.25\\0.2\\0.15\end{pmatrix}$ Forming at least one equation *Dependent on previous M1*

Post-multiplication by transition matrix

5 (i)	$\mathbf{P}=\begin{pmatrix}0.16 & 0.84 & 0 & 0\\ 0.28 & 0 & 0.72 & 0\\ 0.43 & 0 & 0 & 0.57\\ 1 & 0 & 0 & 0\end{pmatrix}$	B2 **2**	*Allow tolerance of* ± 0.0001 *in probabilities throughout this question* Give B1 for two rows correct
(ii)	$(1\quad 0\quad 0\quad 0)\,\mathbf{P}^9$ $=(0.3349\quad 0.3243\quad 0.2231\quad 0.1177)$ $\text{Prob}(C)=0.2231$	M2 A1 **3**	Using $\mathbf{P}^9$ Give M1 for using $\mathbf{P}^{10}$
(iii)	Week 5 $(1\quad 0\quad 0\quad 0)\,\mathbf{P}^4$ $=(0.5020\quad 0.2851\quad 0.1577\quad 0.0552)$	B1 M1 A1 **3**	First row of a power of **P** *SC* Give B0M1A1 for Week 9 and 0.3860 0.3098 0.2066 0.0976
(iv)	$\mathbf{P}^7=\begin{pmatrix}. & 0.2869 & . & .\\ . & . & . & .\\ . & . & . & .\\ . & . & . & .\end{pmatrix}$ $\mathbf{P}^8=\begin{pmatrix}. & . & . & .\\ . & . & 0.2262 & .\\ . & . & . & .\\ . & . & . & .\end{pmatrix}$ Probability is 0.2869×0.2262 $=0.0649$	M1M1 M1 A1 **4**	Elements from $\mathbf{P}^7$ and $\mathbf{P}^8$ Multiplying appropriate probabilities
(v)	Expected run length is $\dfrac{1}{1-0.16}=1.19$ (3 sf)	M1 A1 **2**	Allow 1.2
(vi)	$\mathbf{P}^n \to \begin{pmatrix}0.3585 & 0.3011 & 0.2168 & 0.1236\\ 0.3585 & 0.3011 & 0.2168 & 0.1236\\ 0.3585 & 0.3011 & 0.2168 & 0.1236\\ 0.3585 & 0.3011 & 0.2168 & 0.1236\end{pmatrix}$ *A*: 0.3585 *B*: 0.3011 *C*: 0.2168 *D*: 0.1236	M1 M1 A2 **4**	Evaluating $\mathbf{P}^n$ with $n\geq 10$ *or Obtaining (at least) 3 equations from* $\mathbf{p}\,\mathbf{P}=\mathbf{p}$ Limiting matrix with equal rows *or Solving to obtain one equilib prob* Give A1 for two correct
(vii)	Expected number is 145×0.3585 ≈ 52	M1 A1 ft **2**	
(viii)	$(0.4\quad 0.25\quad 0.2\quad 0.15)\begin{pmatrix}a & 1-a & 0 & 0\\ b & 0 & 1-b & 0\\ c & 0 & 0 & 1-c\\ 1 & 0 & 0 & 0\end{pmatrix}$ $=(0.4\quad 0.25\quad 0.2\quad 0.15)$ $0.4a+0.25b+0.2c+0.15=0.4$ $0.4(1-a)=0.25$ $0.25(1-b)=0.2$ $0.2(1-c)=0.15$ $a=0.375,\quad b=0.2,\quad c=0.25$	M1 A1 M1 A1 **4**	Transition matrix and $(0.4\quad 0.25\quad 0.2\quad 0.15)$ Forming at least one equation *Dependent on previous M1*

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4757 Further Applications of Advanced Mathematics

General comments

The work on this paper was again of a high standard and about 40% of the candidates scored 60 marks or more (out of 72). Many candidates produced substantially correct solutions to all three of their questions. The most popular combination of questions was questions 1, 2 and 4. Overall, questions 1 and 2 were each attempted by about 80% of the candidates, question 4 by about 60% and questions 3 and 5 were each attempted by about 40% of the candidates.

Comments on individual questions

1) *(Vectors)*
There were very many good answers to this question. Most candidates used efficient methods to answer the four parts and applied the techniques competently, but arithmetic and algebraic slips were fairly frequent. In part (i), a common error was to use a scalar product instead of the vector product in the formula $\left|\overrightarrow{\mathbf{AC}}\times\overrightarrow{\mathbf{AB}}\right| / \left|\overrightarrow{\mathbf{AB}}\right|$. In part (iv), the point of intersection was almost always obtained correctly although it was not immediately obvious to all candidates, from the given result in part (ii), that the lines intersect when $p = 5$.

2) *(Multi-variable calculus)*
The partial differentiation was usually done accurately in part (i) then applied correctly to find the normal line in part (ii). Part (iii) was also answered well. In part (iv) it was expected that the partial derivatives at P would be used to find the approximate small change in g. However, most candidates substituted in to obtain g in terms of μ, and could then write down the linear approximation; this was perfectly acceptable, and a similar method could be used in part (vi). Most could see how part (v) followed from part (iv). Part (vi) invited candidates to repeat the work done in parts (iv) and (v) using Q instead of P, but a substantial number were unable to make any progress here.

3) *(Differential geometry)*
Most candidates could find the arc length in part (i) and the curved surface area in part (ii). The method for finding the centre of curvature in part (iii) was generally well understood, but the correct answer was quite rare. As well as arithmetic slips, sign errors were common, particularly going in the wrong direction along the normal. Finding a unit normal vector also caused some difficulty. In part (iv), most candidates were able to find the envelope correctly.

4) *(Groups)*
This question was answered very well indeed, and it was only the final part (viii) which caused any problems. Despite having expressed all the elements of *G* as powers of the generator 2 in part (vi), most candidates were unable to pick out the subgroup of order 6.

5) *(Markov chains)*
This was found to be more difficult than the corresponding question last year. The techniques were generally well understood and calculators were used competently; parts (i), (ii), (vi), (vii) and (viii) were all answered very well. Many candidates were not sufficiently careful in part (iii), for example having found that $\mathbf{P}^4$ is the appropriate power, giving the answer as week 4 instead of week 5. Most could not answer part (iv) correctly; the usual error was to use elements from $\mathbf{P}^7$ and $\mathbf{P}^{15}$ instead of from $\mathbf{P}^7$ and $\mathbf{P}^8$. In part (v), many candidates used the formula $p/(1-p)$ forgetting that, in this case, the first day of the run is to be included in the expected run length.